# Propositional Logic

# ***Logistics:*** Lecture Participation

# Lecture Participation

- Starting Wednesday, we will be using the website PollEV to ask questions in lecture for attendance credit.
- If you answer these questions in lecture, you'll get attendance credit for the day.
  - You don't need to have the right answers – you just need to respond to the questions.
- If you'd prefer not to attend lectures, that's okay! You can opt to count your final exam in place of participation.
  - We'll send out a form where you can opt-out of participation in Week 4.
- CGOE students: We automatically opt you out of participation, since we assume you aren't physically here.

# Lecture Participation

- We'll dry-run PollEV questions today.
- Let's start with the following warm-up:

Make a music recommendation!

Answer at
***https://cs103.stanford.edu/pollev***

Click "Register" and enter your Stanford e-mail to get to the SUNet login page.

- Here are a few music recs of our own:
  - Jami Sieber - *Timeless*.
  - Aaron Parks - *Little Big* and *Little Big II*.
  - Arthur Moon - NPR Music Tiny Desk Concert.
  - Shakey Graves – *Roll the Bones* (check out *Audiotree Live* version).

# Propositional Logic

***Question:*** How do we formalize the definitions and reasoning we use in our proofs?

# Where We're Going

- ***Propositional Logic*** (Today)
  - Reasoning about Boolean values.
- ***First-Order Logic*** (Wednesday/Friday)
  - Reasoning about properties of multiple objects.

# Outline for Today

- ***Propositional Variables***
  - Booleans, math edition!
- ***Propositional Connectives***
  - Linking things together.
- ***Truth Tables***
  - Rigorously defining connectives.
- ***Simplifying Negations***
  - Mechanically computing negations.

# Propositional Logic

$$TakeMath51 \lor TakeCME100$$

$$\neg FirstSucceed \rightarrow TryAgain$$

$$IsCardinal \land IsWhite$$

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These are ***propositional variables***. Each propositional variable stands for a ***proposition***, something that is either true or false.

$$TakeMath51 \lor TakeCME100$$

$$\neg FirstSucceed \rightarrow TryAgain$$

$$IsCardinal \land IsWhite$$

These are ***propositional connectives***, which link propositions into larger propositions

# Propositional Variables

- In propositional logic, individual propositions are represented by ***propositional variables***.

- Each variable can take one one of two values: true or false. You can think of them as `bool` values.

# Propositional Connectives

- There are seven propositional connectives, five of which will be familiar from programming.
- First, there's the logical "NOT" operation:

$$\neg p$$

- You'd read this out loud as "not $p$."
- The fancy name for this operation is ***logical negation***.

# Truth Tables

- A ***truth table*** is a table showing the truth value of a propositional logic formula as a function of its inputs.
- Let's examine the truth tables for the connectives we're exploring today!

“I don’t love cupcakes.”

“I don’t love cupcakes.”

***LoveCupcakes*** : I love cupcakes.

“I don’t love cupcakes.”

***LoveCupcakes*** : I love cupcakes.

¬***LoveCupcakes***

# Propositional Variables

- In propositional logic, individual propositions are represented by ***propositional variables***.
- Each variable can take one one of two values: true or false. You can think of them as `bool` values.
- In a move that contravenes programming style conventions, propositional variables are usually represented as lower-case letters, such as *p*, *q*, *r*, *s*, etc.
  - That said, there's nothing stopping you from using multiletter names!

“I don’t love cupcakes.”

***LoveCupcakes*** : I love cupcakes.

¬***LoveCupcakes***

“I don’t love cupcakes.”

***c*** : I love cupcakes.

$\neg$***LoveCupcakes***

“I don’t love cupcakes.”

$c$ : I love cupcakes.

$\neg c$

# Propositional Connectives

- There are seven propositional connectives, five of which will be familiar from programming.
- Next, there's the logical "AND" operation:

$$p \wedge q$$

- You'd read this out loud as "$p$ and $q$."
- The fancy name for this operation is ***logical conjunction***.

“It’s cardinal and white.”

“It’s cardinal and white.”

***IsCardinal*** : It’s cardinal.

“It’s cardinal and white.”

***IsCardinal*** : It’s cardinal.

***IsWhite*** : It’s white.

“It’s cardinal and white.”

***IsCardinal*** : It’s cardinal.

***IsWhite*** : It’s white.

***IsCardinal*** ∧ ***IsWhite***

“It’s cardinal and white.”

***p*** : It’s cardinal.

***q*** : It’s white.

***IsCardinal*** ∧ ***IsWhite***

“It’s cardinal and white.”

***p*** : It’s cardinal.

***q*** : It’s white.

**p** ∧ ***q***

# Propositional Connectives

- There are seven propositional connectives, five of which will be familiar from programming.
- Then, there's the logical "OR" operation:

$$p \vee q$$

- You'd read this out loud as "*p* or *q*."
- The fancy name for this operation is ***logical disjunction***. This is an *inclusive* or.

“You must take Math 51 or CME 100.”

“You must take Math 51 or CME 100.”

***TakeMath51*** : You must take Math 51.

“You must take Math 51 or CME 100.”

***TakeMath51*** : You must take Math 51.

***TakeCME100*** : You must take CME 100.

“You must take Math 51 or CME 100.”

***TakeMath51*** : You must take Math 51.

***TakeCME100*** : You must take CME 100.

***TakeMath51*** ∨ ***TakeCME100***

“You must take Math 51 or CME 100.”

***TakeMath51*** : You must take Math 51.

***TakeCME100*** : You must take CME 100.

***TakeMath51*** ∨ ***TakeCME100***

These are ***propositional variables***. Each propositional variable stands for a ***proposition***, something that is either true or false.

“You must take Math 51 or CME 100.”

***TakeMath51*** : You must take Math 51.

***TakeCME100*** : You must take CME 100.

***TakeMath51*** ∨ ***TakeCME100***

This is a ***propositional connective***, which links propositions into larger propositions

“You must take Math 51 or CME 100.”

***TakeMath51*** : You must take Math 51.

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***TakeMath51*** ∨ ***TakeCME100***

“You must take Math 51 or CME 100.”

***p*** : You must take Math 51.

***q*** : You must take CME 100.

***TakeMath51*** ∨ ***TakeCME100***

“You must take Math 51 or CME 100.”

***p*** : You must take Math 51.

***q*** : You must take CME 100.

$p \vee q$

# Propositional Connectives

- There are seven propositional connectives, five of which will be familiar from programming.
- There's also the "truth" connective:

$$\top$$

- You'd read this out loud as "true."
- Although this is technically considered a connective, it "connects" zero things and behaves like a variable that's always true.

# Propositional Connectives

- There are seven propositional connectives, five of which will be familiar from programming.
- Finally, there's the "false" connective.

- You'd read this out loud as "false."
- Like $\top$, this is technically a connective, but acts like a variable that's always false.

# Inclusive and Exclusive OR

- The ∨ connective is an *inclusive* "or." It's true if at least one of the operands is true.
  - It's similar to the `||` operator in C, C++, Java, etc. and the `or` operator in Python.
- Sometimes we need an *exclusive* "or," which isn't true if both inputs are true.
- We can build this out of what we already have.

Write a propositional logic formula for the exclusive OR of $p$ and $q$.

Answer at
***https://cs103.stanford.edu/pollev***

***Quick Question:***

What would I have to show you to convince you that the statement $p \wedge q$ is false?

***Quick Question:***

What would I have to show you to convince you that the statement $p \vee q$ is false?

# de Morgan's Laws

$\neg(p \wedge q)$ ***is equivalent to*** $\neg p \vee \neg q$

$\neg(p \vee q)$ ***is equivalent to*** $\neg p \wedge \neg q$

# de Morgan’s Laws in Code

- ***Pro tip:*** Don't write this:

```
if (!(p() && q())) {
    /* … */
}
```

- Write this instead:

```
if (!p() || !q()) {
    /* … */
}
```

- (This even short-circuits correctly: if `p()` returns false, `q()` is never evaluated.)

# Mathematical Implication

# Implication

- We can represent implications using this connective:

$$\boldsymbol{p \rightarrow q}$$

- You'd read this out loud as "$p$ implies $q$."
  - The fancy name for this is the ***material conditional***.
- ***Question:*** What should the truth table for $p \rightarrow q$ look like?

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| F | F | |
| F | T | |
| T | F | |
| T | T | |

How should we fill in these blanks?

Answer at
***https://cs103.stanford.edu/pollev***

The ***pig*** does the thing.

Sean throws ***qookies***.

Contract upheld?

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| | | |

## ***A Contract (from Friday):***

If a flying pig bursts into the room and sings a pitch-perfect version of the national anthem, then Sean will throw cookies to the class.

Sean throws ***qookies***.

The ***pig*** does the thing.

Contract upheld?

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | T |

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|---|---|---|
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| F | T | T |
| T | T | T |

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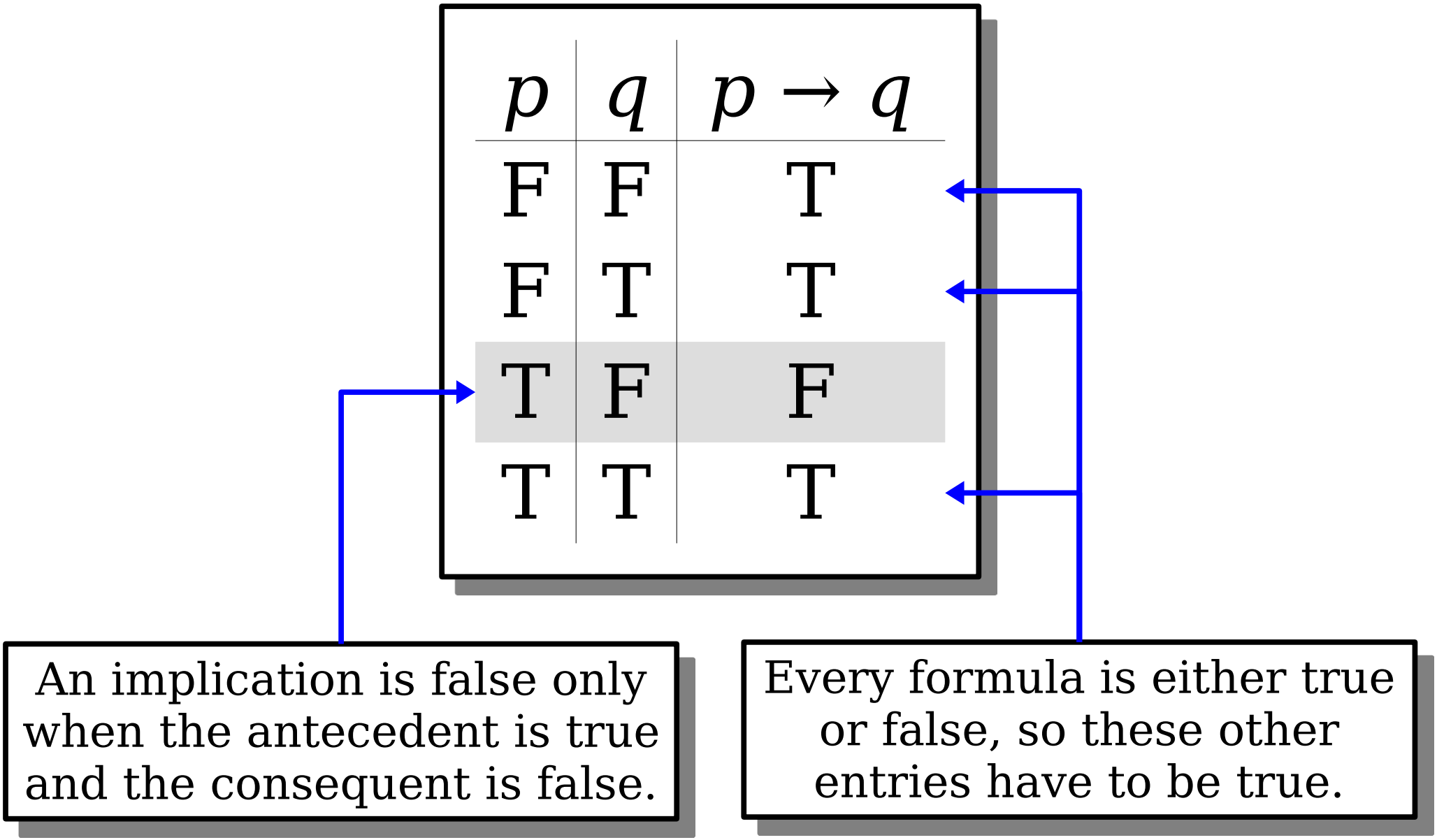
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

## ***A Contract (from Friday):***

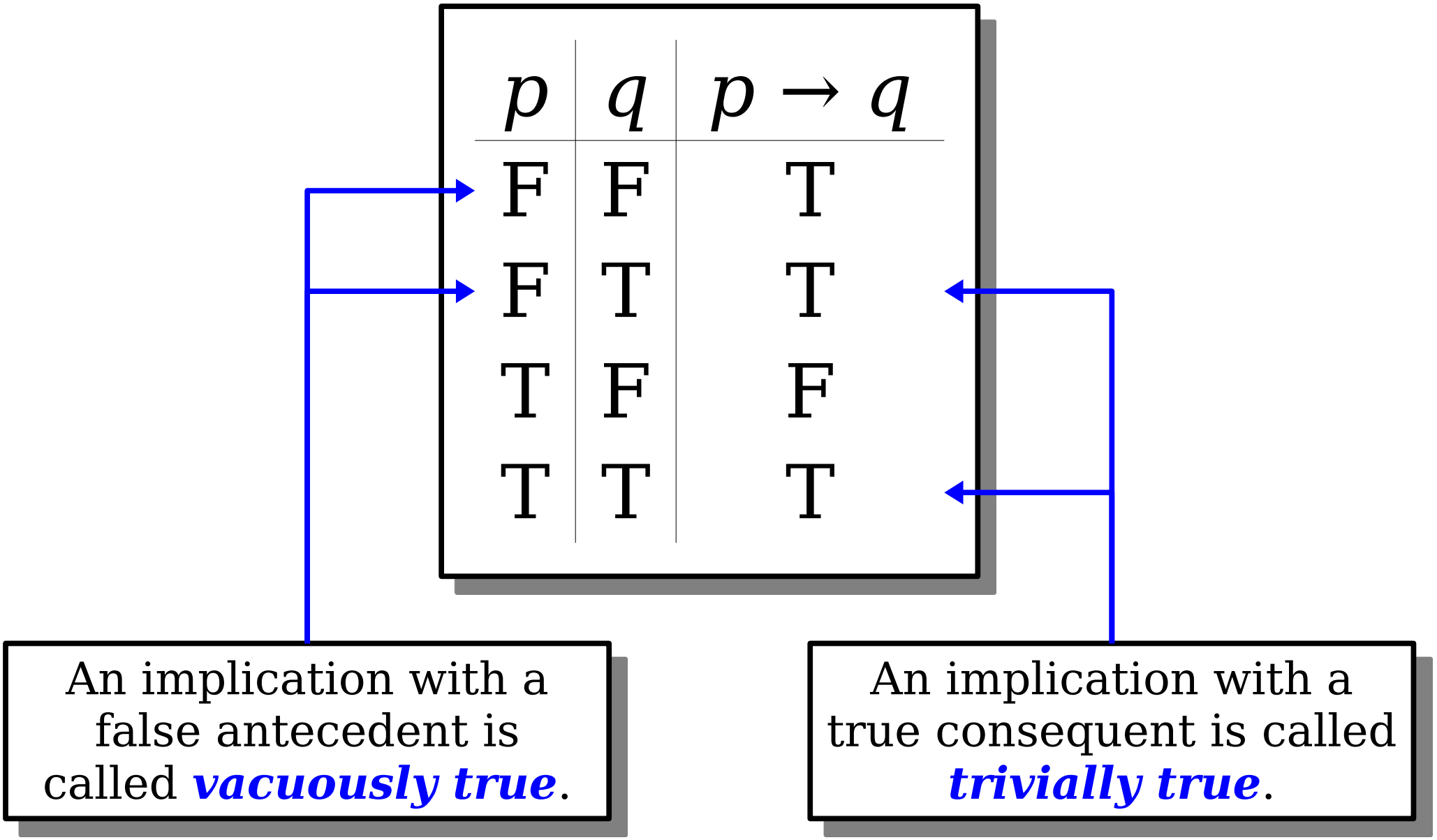
If a flying pig bursts into the room and sings a pitch-perfect version of the national anthem, then Sean will throw cookies to the class.

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |


$p$ $q$ $p \rightarrow q$
F F T
F T T
T F F
T T T
An implication is false only when the antecedent is true and the consequent is false.
Every formula is either true or false, so these other entries have to be true.

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

***Important observation:***
The statement $p \to q$ is true whenever $p \wedge \neg q$ is false.


$p$ $q$ $p \rightarrow q$
F F T
F T T
T F F
T T T
An implication with a false antecedent is called ***vacuously true***.
An implication with a true consequent is called ***trivially true***.

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

***Please commit this table to memory***. We're going to need it, extensively, over the next couple of weeks.

“If at first you don’t succeed, try again.”

“If at first you don’t succeed, try again.”

***FirstSucceed*** : You succeed at first.

“If at first you don’t succeed, try again.”

***FirstSucceed*** : You succeed at first.

***TryAgain*** : You ought to try again.

“If at first you don’t succeed, try again.”

***FirstSucceed*** : You succeed at first.

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$\neg$***FirstSucceed*** $\rightarrow$ ***TryAgain***

“If at first you don’t succeed, try again.”

***p*** : You succeed at first.

***q*** : You ought to try again.

$\neg$***FirstSucceed*** $\rightarrow$ ***TryAgain***

“If at first you don’t succeed, try again.”

$p$ : You succeed at first.

$q$ : You ought to try again.

$$\neg p \rightarrow q$$


IF IT AIN'T FRESHLY SLICED,
IT AIN'T
SINCE 1956
Jersey Mike's
SUBS


IF IT AIN'T FRESHLY SLICED,
IT AIN'T
SINCE 1956
Jersey Mike's
SUBS


***JerseyMikes*** : It’s Jersey Mike’s.


***JerseyMikes*** : It’s Jersey Mike’s.

***FreshlySliced*** : It’s freshly sliced.


***JerseyMikes*** : It's Jersey Mike's.

***FreshlySliced*** : It's freshly sliced.

$\neg$***FreshlySliced*** $\rightarrow$ $\neg$***JerseyMikes***


***JerseyMikes*** : It's Jersey Mike's.

***FreshlySliced*** : It's freshly sliced.

$\neg$***FreshlySliced*** $\rightarrow$ $\neg$***JerseyMikes***

***JerseyMikes*** $\rightarrow$ ***FreshlySliced***

# An Important Equivalence

- The truth table for for $p \rightarrow q$ is chosen so that the following is true:

$$p \rightarrow q \quad \textit{is equivalent to} \quad \neg(p \wedge \neg q)$$

- Later on, this equivalence will be incredibly useful:

$$\neg(p \rightarrow q) \quad \textit{is equivalent to} \quad p \wedge \neg q$$

# Side Note: Contrapositive

We can use truth tables to demonstrate the equivalence of $p \to q$ and $\neg q \to \neg p$.

| $p$ | $q$ | $p \to q$ | $\neg p$ | $\neg q$ | |
|---|---|---|---|---|---|
| F | F | T | T | T | |
| F | T | T | T | F | |
| T | F | F | F | T | |
| T | T | T | F | F | |

# Side Note: Contrapositive

We can use truth tables to demonstrate the equivalence of $p \to q$ and $\neg q \to \neg p$.

| $p$ | $q$ | $p \to q$ | $\neg q$ | $\neg p$ | $\neg q \to \neg p$ |
|---|---|---|---|---|---|
| F | F | T | T | T | T |
| F | T | T | F | T | T |
| T | F | F | T | F | F |
| T | T | T | F | F | T |

same :)

# The Biconditional Connective

# The Biconditional Connective

- In our previous lecture, we saw that the statement "$p$ if and only if $q$" means both that $p \rightarrow q$ and $q \rightarrow p$.
- We can write this in propositional logic using the ***biconditional*** connective:

$$\boldsymbol{p \leftrightarrow q}$$

- This connective's truth table has the same meaning as "$p$ implies $q$ and $q$ implies $p$."
- Based on that, what should its truth table look like?

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| F | F | ____ |
| F | T | ____ |
| T | F | ____ |
| T | T | ____ |

How should we fill in these blanks?

Answer at
***https://cs103.stanford.edu/pollev***

# Biconditionals

- The biconditional connective $p \leftrightarrow q$ has the same truth table as $(p \rightarrow q) \wedge (q \rightarrow p)$.
- Here's what that looks like:

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| F | F | T |
| F | T | F |
| T | F | F |
| T | T | T |

# Biconditionals

- The biconditional connective $p \leftrightarrow q$ has the same truth table as $(p \rightarrow q) \wedge (q \rightarrow p)$.
- Here's what that looks like:

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| F | F | T |
| F | T | F |
| T | F | F |
| T | T | T |

One interpretation of ↔ is to think of it as equality: the two propositions must have equal truth values.

# Negating a Biconditional

- How do we simplify

$$\neg(p \leftrightarrow q)$$

  using the tools we've seen so far?
- There are many options, but here are our two favorites:

$$p \leftrightarrow \neg q \qquad\qquad \neg p \leftrightarrow q$$

Question to ponder: what is the truth table for these statements, and where have you seen it before?

# Operator Precedence

- How do we parse this statement?

$$\neg x \to y \vee z \to x \vee y \wedge z$$

- Operator precedence for propositional logic:

$\neg$

$\wedge$

$\vee$

$\to$

$\leftrightarrow$

- All operators are right-associative.
- We can use parentheses to disambiguate.

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- All operators are right-associative.
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# Operator Precedence

- How do we parse this statement?

$$(\neg x) \rightarrow y \vee z \rightarrow x \vee y \wedge z$$

- Operator precedence for propositional logic:

$\neg$

$\wedge$

$\vee$

$\rightarrow$

$\leftrightarrow$

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# Operator Precedence

- How do we parse this statement?

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- Operator precedence for propositional logic:

$\neg$

$\wedge$

$\vee$

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$\leftrightarrow$

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# Operator Precedence

- How do we parse this statement?

$$(\neg x) \to y \vee z \to x \vee (y \wedge z)$$

- Operator precedence for propositional logic:

$\neg$

$\wedge$

$\vee$

$\to$

$\leftrightarrow$

- All operators are right-associative.
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# Operator Precedence

- How do we parse this statement?

$$(\neg x) \to y \vee z \to x \vee (y \wedge z)$$

- Operator precedence for propositional logic:

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$\wedge$

$\vee$

$\to$

$\leftrightarrow$

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# Operator Precedence

- How do we parse this statement?

$$(\neg x) \rightarrow (y \vee z) \rightarrow (x \vee (y \wedge z))$$

- Operator precedence for propositional logic:

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$\vee$

$\rightarrow$

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$$(\neg x) \rightarrow (y \vee z) \rightarrow (x \vee (y \wedge z))$$

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$$(\neg x) \rightarrow ((y \vee z) \rightarrow (x \vee (y \wedge z)))$$

- Operator precedence for propositional logic:

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- Operator precedence for propositional logic:

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$\wedge$

$\vee$

$\rightarrow$

$\leftrightarrow$

- All operators are right-associative.
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# Operator Precedence

- The main points to remember:
  - ¬ binds to whatever immediately follows it.
  - ∧ and ∨ bind more tightly than →.
- We will commonly write expressions like $p \wedge q \rightarrow r$ without adding parentheses.
- For more complex expressions, we'll try to add parentheses.
- Confused? ***Please ask!***

# The Big Table

| Connective | Read Aloud As | C++ Version | Fancy Name | Negation |
|---|---|---|---|---|
| $\neg p$ | “not” | `!` | Negation | $p$ |
| $p \wedge q$ | “and” | `&&` | Conjunction | $\neg p \vee \neg q$<br>$p \rightarrow \neg q$ |
| $p \vee q$ | “or” | `\|\|` | Disjunction | $\neg p \wedge \neg q$ |
| $\top$ | “true” | `true` | Truth | $\bot$ |
| $\bot$ | “false” | `false` | Falsity | $\top$ |
| $p \rightarrow q$ | “implies” | *see PS2!* | Implication | $p \wedge \neg q$ |
| $p \leftrightarrow q$ | “if and only if” | *see PS2!* | Biconditional | $p \leftrightarrow \neg q$<br>$\neg p \leftrightarrow q$ |

Time-Out for Announcements!

# Submitting Work

- All assignments should be submitted through GradeScope.
  - The programming portion of the assignment is submitted separately from the written component.
  - The written component ***must*** be typed; handwritten solutions don't scan well and get mangled in GradeScope.
- All assignments are due at 1:00PM. You have three "late days" you can use throughout the quarter. Each automagically extends assignment deadlines from Friday at 1:00PM to Saturday at 1:00PM; at most one late day can be used per assignment.
  - ***Very good idea:*** Leave at least two hours buffer time for your first assignment submission, just in case something goes wrong.
  - ***Very bad idea:*** Wait until the last minute to submit.
- Your score on the problem sets is the square root of your raw score. So an 81% maps to a 90%, a 50% maps to a 71%, etc. This gives a huge boost even if you need to turn something in that isn't done.

# Office Hours

- Office hours have started (as of Sunday)! Think of them as “drop-in help hours” where you can ask questions on problem sets, lecture topics, etc.
  - Check the Guide to Office Hours on the course website for the schedule.
- TA office hours are held in person in the Huang basement. Keith’s are in Durand 317. Sean’s are in Durand 331-B.
- Once you arrive, sign up on QueueStatus so that we can help people in the order they arrived:

  ***https://queuestatus.com/queues/782***

- Office hours are *much* less crowded earlier in the week than later. Stop by on Sunday, Monday, and Tuesday!

Back to CS103!

# Recap So Far

- A ***propositional variable*** is a variable that is either true or false.
- The ***propositional connectives*** are
  - Negation: $\neg p$
  - Conjunction: $p \wedge q$
  - Disjunction: $p \vee q$
  - Truth: $\top$
  - Falsity: $\bot$
  - Implication: $p \to q$
  - Biconditional: $p \leftrightarrow q$

# Why All This Matters

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- Suppose we want to prove the following statement:

  "If $x + y = 16$, then $x \geq 8$ or $y \geq 8$"

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$$x + y = 16 \rightarrow x \geq 8 \vee y \geq 8$$

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# Why All This Matters

- Suppose we want to prove the following statement:

  "If $x + y = 16$, then $x \geq 8$ or $y \geq 8$"

  $$\neg(x \geq 8 \vee y \geq 8) \rightarrow \neg(x + y = 16)$$

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  "If $x + y = 16$, then $x \geq 8$ or $y \geq 8$"

  $\neg(x \geq 8 \vee y \geq 8) \rightarrow \neg(x + y = 16)$

# Why All This Matters

- Suppose we want to prove the following statement:

"If $x + y = 16$, then $x \geq 8$ or $y \geq 8$"

$\neg(x \geq 8 \vee y \geq 8) \rightarrow x + y \neq 16$

# Why All This Matters

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"If $x + y = 16$, then $x \geq 8$ or $y \geq 8$"

$$\neg(x \geq 8) \wedge \neg(y \geq 8) \rightarrow x + y \neq 16$$

# Why All This Matters

- Suppose we want to prove the following statement:

  "If $x + y = 16$, then $x \geq 8$ or $y \geq 8$"

  $\neg(x \geq 8) \wedge \neg(y \geq 8) \rightarrow x + y \neq 16$

# Why All This Matters

- Suppose we want to prove the following statement:

"If $x + y = 16$, then $x \geq 8$ or $y \geq 8$"

$x < 8 \wedge \neg(y \geq 8) \rightarrow x + y \neq 16$

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"If $x < 8$ and $y < 8$, then $x + y \neq 16$"

***Theorem:*** If $x + y = 16$, then $x \geq 8$ or $y \geq 8$.

***Proof:*** We will prove the contrapositive, namely, that if $x < 8$ and $y < 8$, then $x + y \neq 16$.

Pick $x$ and $y$ where $x < 8$ and $y < 8$. We want to show that $x + y \neq 16$. To see this, note that

$$\begin{aligned} x + y &< 8 + y \\ &< 8 + 8 \\ &= 16. \end{aligned}$$

This means that $x + y < 16$, so $x + y \neq 16$, which is what we needed to show. ■

# Why This Matters

- Propositional logic lets us symbolically manipulate statements and theorems.
  - This can help us better understand what a theorem says or what a definition means.
- It’s also very useful for proofs by contradiction and contrapositive.
- Being able to negate statements mechanically can reduce the likelihood of taking an negation of contrapositive wrong.

# Negation Practice

- Here's a propositional formula that contains some negations. Simplify it as much as possible:

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# Next Time

- ***First-Order Logic***
  - Reasoning about groups of objects.
- ***First-Order Translations***
  - Expressing yourself in symbolic math!